A Novel Point Based Non-rigid Registration Method and Its Application on Brain Shift^{*}

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ABSTRACT

This paper presents a novel point based non-rigid registration (NRR) method. We overcome the limitations of the traditional point-based NRR methods by using a biomechanical model with a Robust Point Matching framework. Our new framework relies on robust regression technique to handle *sparse* point sets which are *partially correlated*. The non-rigid registration problem is formulated as a two variables (Correspondence and Mapping) functional minimization problem. The functional is decomposed into stress and similarity components. Linear Finite Element method using a patient-specific biomechanical model along with Expectation Maximization method are used for the simultaneous computation/approximation of both Correspondence and Mapping function. Gaussian distribution-based search range is used to improve the computational efficiency and robustness of the method. The combination of the search range with Least Trimmed Squared can effectively detect outliers in both source and target point sets. Preliminary performance data from an experimental evaluation for brain shift compensation shows the effectiveness of this method for *sparse* and even *partially correlated* source and target point sets.

Keywords: Non-rigid registration, Outliers, Expectation and Maximization, Biomechanical model, Robust regression, Brain shift

1. INTRODUCTION

In this paper we focus on point based NRR methods.¹ Point based non-rigid registration (NRR) methods compute a mapping function F from given source point set S to target point set T with/without correspondence C. We classify the point based NRR methods into two categories, based on the order we compute the correspondence and mapping function. The *Type 1* methods first find the correspondence between the point sets and then solve for the mapping function.^{2–4} While, the *Type 2* methods simultaneously solve for both the correspondence and the mapping function.^{5–7} Our new approach is a *Type 2* point based NRR method. Specifically, we modify the Robust Point Matching (RPM) method, which is an extension of the well-known Iterative Closest Point (ICP) algorithm.⁸ The traditional RPM method employs Thin-Plate Splines (TPS) to approximate the mapping function.

The basis function of TPS is a solution of the biharmonic equation⁹ and does not have compact support. Therefore, for real applications, RPM registration with TPS lead to unrealistic deformation far from the given point set. We illustrate this point in Figure 1 using a landmark based NRR based on ITK ThinPlateSplineKernelTransform.¹⁰ Notice, that this figure shows only the points of the selected slice, there are additional red and green landmark points in the rest of the volume. Our ITK NRR software registers iMRI with blood-oxygen-level dependent (BOLD) image based on the feature or landmark points selected by surgeons. Selection of landmarks (red points in Figure 1(a) and Figure 1(b)) near by the craniotomy leads into an unrealistic deformation of the region far from the area of selected points (see Figure 1(c)). However, if we use additional feature points (i.e., green points in Figure 1(a) and Figure 1(b)), the deformation is more realistic (see Figure 1(d)) but still not very accurate (see Figure 1(e)).

Other groups also realized this problem for TPS. Yang et al.⁶ and Wachowiak et al.¹¹ provided compact support radius basis function (CSRBF) to overcome this difficulty and Papademetris et al.⁷ replaced TPS with

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Figure 1. Landmark features in the same slice of an iMRI (a) and Blood-Oxygen-Level Dependent image (b) used with Thin-Plate Splines with points selected near by the craniotomy (c) and both near and far away from the craniotomy (d) are compared with Free-Form Deformation (e).

Free-Form Deformation $(FFD)^{12}$ as the mapping function. In comparison to TPS functions, FFD and CSRBF affect local deformation, but at the same time it will limit the estimated deformation to be valid only near the point sets. Figure 1(e) shows that the control point a(4, 4) of FFD only influences its 4×4 neighborhood denoted as blue color points. In another words, if we want to use FFD or CSRBF to estimate the deformation of the entire brain, we need the point set to be dense or cover the whole brain. However, in some real applications, only sparse information is available. For instance, to compensate for brain shift in Image-guided Neurosurgery (IGNS), we need to estimate the deformation of the entire brain based on scanned cortical surface.^{13, 14}

In this paper we overcome this difficulty (i.e., rely on a priori knowledge to estimate the deformation given the sparse point sets) by combining (see Section 3.1) the physics based approach in³ with the RPM framework, where the stress energy is used as a regularization term. This method is capable to estimate the deformation far from the point sets because the biomechanical model agrees well with the behavior of the brain only with sparse information (boundary condition) available. Furthermore, we extend RPM to deal with the outliers in both source and target point sets (see Section 3.3). This means that this method still works even for given partially correlated point sets. However, these extensions result in a more complicate and difficult problem to solve. Unlike TPS based method, whose analytical solution is available, we have to use finite element methods to find the numerical solution. Finally, in Section 4, we present preliminary results that combine the patientspecific biomechanical finite element model³ with our robust regression method to get more realistic deformation everywhere. In summary, the contribution of this paper is a novel point based NRR method, for sparse and even partially correlated point sets, that estimates realistic mapping function for the whole domain.

2. RELATED WORK

There are several algorithms to compute correspondence for Type 1 methods. Clatz et al.,³ to compensate for brain shift, used block matching to find the correspondence. DeLorenzo et al.⁴ used game theory to track cortical surface in order to find the correspondence. Ferrant et al.² simulated the extracted surface of the ventricle as a membrane to perform tracking. The applicability of these methods depends on how easy or effective is to determine correspondence. In contrast, Type 2 methods are independent the way initial correspondence is determined and thus are more flexible. However, these methods are more difficult because one has to determine both F and C simultaneously.

A well-known Type 2 method is the iterative closest point (ICP).⁸ ICP utilizes the nearest-neighbor relationship to estimate the correspondence and then iteratively refines the transformation based on the correspondence. ICP is fast and can be guaranteed to converge to a local minimum. However, it uses binary correspondence and supports only rigid mappings. In⁵ Chui et al. overcame these drawbacks by using a Robust Point Matching (RPM) algorithm. The algorithm is based on softassign, for the correspondence, and Thin-Plate Splines for the non-rigid mapping.

In addition, Miga. et al. employed RPM for non-rigid registration,¹⁵ too. They identified vessels in both pre-operative MRI and Laser Range Scanning image, and then used RPM to force the corresponding vessels to exactly match each other under the constraint of bending energy, for the whole image. Li et al.¹⁶ employed this method for NRR for longitudinal breast MR image. Papademetris et al.⁷ extend RPM to improve its ability to

deal with larger point sets and partially correlated point sets. The Yale group applied this method in order to form composite activation maps from functional MRI (fMRI). They were able to demonstrate that RPM-based methods are superior to pure intensity based registration for fusiform gyrus.

3. METHOD

To solve the mapping function and correspondence, the NRR problem is formulated as a functional minimization decomposed into a regularization energy and a similarity energy.

3.1 Energy function

Suppose there are two point sets: a source point set, $S = \{s_i\}_{i=1}^p$ and a Target point set $T = \{t_i\}_{i=1}^l$ the functional is constructed as follows:

$$W(u,C) = \int_{\Omega} \sigma(u)^t \epsilon(u) + \lambda \sum_{i=1}^p \|s_i + u(s_i) - \sum_{t_j \in \Omega_R} c_{ij} t_j \|^2$$
(1)

The first term is the regularization energy defined by the stress energy of the linear elastic model and the second term is the similarity energy. The coefficient λ is used to control the trade-off between these two energies. We make the estimation of the mapping function more realistic by using the stress energy as the regularization term. Our basis for comparison is the work in,^{5–7} which use the smoothing measure of TPS or CSRBF as the regularization term.

In the similarity energy, Ω_R defines the search range, which is a sphere centered at the source point with radius *R*. c_{ij} is the probability with which the point s_i corresponds with t_j located in Ω_R . *u* is the unknown displacement field and *C* is unknown correspondence matrix with entry c_{ij} . Correspondence matrix *C* is similar with that in,⁵ but we define a range Ω_R and only take into account the target points located in Ω_R . The search range basically makes our NRR act as a multi-resolution registration. As the range reduces, the registration will go from the coarse level to the fine level. c_{ij} is calculated as equation 2. For each source point s_i , assume its potential correspondences are subject to the Gaussian distribution,

$$c_{ij} = \frac{c'_{ij}}{\sum_{k=1}^{k=m} c'_{ik}}, \quad c'_{ij} = \frac{1}{R\sqrt{2\pi}} e^{\frac{-(t_j - s_i)^2}{2R^2}}, \forall t_j \in \Omega_R, j = 1 \dots m$$
(2)

Combined the search range with the Least Trimmed Squares (LTS) robust regression technique,¹⁷ we can effectively detect the outliers existing in both point sets.

It is difficult to find the analytical solution from equation 1. We use finite element method to discretize the problem by approximating

$$u = \sum_{i=0}^{i=n} N_i U_i \tag{3}$$

where n is the number of the vertices of the finite element mesh. N is the shape function and U is node displacement vector. For simplicity, define vector D with entry $D_i(c_{ij}) = s_i - \sum_{t_j \in \Omega_R} c_{ij}t_j$ and equation 1 can be discretized as,

$$W(U,C) = U^T K U + \lambda (HU - D(C))^T (HU - D(C))$$

$$\tag{4}$$

K is the stiffness matrix of size $3n \times 3n$. The building of K has been well documented in.¹⁸ H is the linear interpolation matrix of size $3p \times 3n$.

Each registration point o_k with number k contained in tetrahedron with vertex number $c_i, i \in [0:3]$ has contribution to four 3×3 submatrices: $[H]_{kc_0}, [H]_{kc_1}, [H]_{kc_2}, [H]_{kc_3}$. $[H]_{kc_i}$ is defined as: $[H]_{kc_i} = diag(h_i, h_i, h_i)$. The linear interpolation factor h_i is calculated as:

$$\begin{bmatrix} h_0\\ h_1\\ h_2\\ h_3 \end{bmatrix} = \begin{bmatrix} v_{c_0}^x & v_{c_1}^x & v_{c_2}^x & v_{c_3}^x\\ v_{c_0}^y & v_{c_1}^y & v_{c_2}^y & v_{c_3}^y\\ v_{c_0}^z & v_{c_1}^z & v_{c_2}^z & v_{c_3}^z\\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} o_k^x\\ o_k^y\\ o_k^z\\ 1 \end{bmatrix}$$
(5)

where v_{c_i} is the vertex with number c_i .

Similar to,³ the equation 4 can be solved by:

$$\frac{\partial W}{\partial U} = [K + H^T H]U - H^T D(C) = 0 \Rightarrow [K + H^T H]U = H^T D(C)$$
(6)

Regularization of the solution using the mechanical energy unavoidably contains approximation error.³ We address this problem by the iterative estimation of the displacement:

$$F_0 = 0, \ F_{i-1} = KU_{i-1},$$

[K + H^TH]U_i = H^TD(C) + F_{i-1} (7)

 $K + H^T H$ is semi-positive definite matrix, therefore we use Conjugate Gradient (CG)¹⁹ to resolve the linear system of equations. This component is computed in parallel, facilitated by PETSc implementation.²⁰

This energy function has two unknowns: U and C. If one is closer to the real solution and the other is too. So Expectation and Maximization is employed to solve them.

3.2 Expectation and Maximization

The Expectation and Maximization (EM) algorithm²¹ is a general algorithm for maximum-likelihood²² estimation of the model parameter (unknowns) in the presence of missing or hidden data. To estimate the model parameters, EM proceeds iteratively and each iteration of the EM algorithm consists of two steps: The E step and the M step. In the E step, the missing data are estimated given the observed data and current estimate of the model parameters. In the M step, the likelihood function is maximized under the assumption that the missing data are known. The estimate of the missing data from the E step are used in lieu of the actual missing data. Convergence is assured since the algorithm is guaranteed to increase the likelihood at each iteration.

The intuition behind EM is: alternate between estimating the unknowns and the missing data. The point based non-rigid registration can be stated as: Find the mapping function (unknown) between the source point set and target point set in the absence of the correspondence (missing data). The EM proceeds as follows. E-step: estimate correspondence given current estimate of the mapping function according to equation 2. M Step: calculate mapping function given correspondence according to equation 6.

3.3 Outlier rejection

We present an outlier detection technique by combining the search range with Least Trimmed Square (LTS) estimator.¹⁷ LTS estimator is a robust regression technique tolerant to outliers. Considering a linear regression model for sample (\boldsymbol{x}_i, y_i) with a response variable y_i and a vector of p explanatory variables \boldsymbol{x}_i :

$$y_i = \beta \mathbf{x}_i + \varepsilon_i, i = 1, \dots, n.$$
(8)

where β is the coefficient vector and ε is a random error term.

The LTS estimator is defined as:

$$\overline{\beta} = \underset{\beta \in \Re^{P}}{\operatorname{argmin}} \sum_{i=1}^{h} r_{i}^{2}(\beta)$$
(9)

where $r_i^2 \leq \ldots \leq r_n^2$ are the ordered squared residuals. Equation 9 is very similar to traditional least square with the only difference that only h observations with the smallest squared residuals are used in the summation, thereby allowing the fit to stay away from the outliers. The best robustness properties are achieved when h, termed as trimming constant, is approximately n/2, in which case the breakdown point attains 50%.¹⁷ To determine the LTS estimator, we need to examine the total of $\binom{n}{h}$ subsamples with size h, and thus, the computation is very slow if n is large.²³

In this paper, combined with search range R, we present an approximation method. This method contains two steps: trial step and outlier rejection step.

trial step: Using EM algorithm to find the mapping function corresponding to the search range R and then transform the source points. The purpose of this step is not the mapping function due to its bias induced by the outliers, but the detection of the outliers in the next step.

Outlier rejection step: based on the transformed source point set, for each source point, find target points within the search range $R = R \times a$, where a is the annealing parameter and is equal to 0.93 as suggested in.⁵ If there are no target points for this source point, mark it as outlier. Replace the original source point set with this marked source point and estimate mapping function again. The difference between this modified LTS and the traditional LTS is that we use the search range instead of h to perform outlier rejection, and therefore no need for the ordering of the residuals. For the outliers in the target point set, they do not involve the computation if they are out of the range. So, this method can be used to deal with the outliers in both point sets.

Algorithm 1 describes this EM procedures embedded with LTS. It has two loops. The first one is EM loop for the correspondence and the second one is approximation to interpolation loop for the increase of the accuracy of the solution.

Algorithm 1 Point based NRR
Input: S: source point set, T: target point set, R: search range, a: annealing parameter, λ : trade-off parameter,
ϵ : tolerance
Output: U: displacement vector
1: Initialize R, a, λ and ϵ
2: repeat
3: LTS trial step:
4: E step: Estimate correspondence C according to equation 2
5: M step: Solve U according to equation 6
6: Transform S based on U: $S \leftarrow U(S)$
7: LTS outlier rejection step:
8: $S \leftarrow S - s_i$ if there are no target points in $\Omega_{R \times a}$
9: recalculate U based on outliers rejected S
10: $error \leftarrow U_i - U_{i-1} $ between successive iterations
11: $R \leftarrow R \times a$
12: until $error < \epsilon$
13: repeat
14: $F_i \leftarrow KU_i$
15: $U_{i+1} \leftarrow [K + H^T H]^{-1} [H^T D(C) + F_i]$
16: until Convergence
17: Output U

4. EXPERIMENTAL EVALUATION

Brain shift significantly compromises the fidelity of the IGNS. There are many papers in the literature addressing this issue.^{3,13,14,24-26} According to the devices used in the Operating Room (OR) to obtain the intra-operative data, we classify them into two categories: surface method and volume method. The surface method introduces cameras¹³ or laser ranger scanner (LRS)^{14,24,25} into the OR to obtain the deformed cortical surface and the volume method uses iMR,³ iUS²⁶ to obtain the deformed volume. Recently, iCT is employed by Archip et al.²⁷

and Elhawary et al.²⁸ to deal with the navigated tumor ablation of the liver. Although Eggers et al.²⁹ used iCT for image-guided cranial surgery, they only focused on the rigid registration. The reports about using iCT for the brain shift have not been seen up to now.

For the consideration of the evaluation of our NRR, we concentrate on the application of our NRR technique for the surface method, which is characterized by:

- only sparse intra-operative information available (scanned surface)
- outliers in both point sets
- entire brain deformation is required

To obtain the deformed cortical surface, we introduce LRS of 3D Digital Corporation³⁰ into the OR. Its specifications are shown in Table 1. LRS can be tracked by attaching four tracked balls on the top of it as shown in Figure 2(a).

Table 1. LRS Specification				
Resolution	Standard Deviation	Depth of Field	Point Set Density	
175 microns	± 20 microns	300 - 900 mm	Up to 1000×1000	

The optimal Depth of Field is about 400mm, to the best of our knowledge, which means the optimal distance of the LRS from the exposed cortical surface is 400mm. Figure 2(b) shows the position of the LRS in the OR and Figure 2(c) shows a laser scanning from the surface. The initial surface is acquired by extracting the surface from the mesh. The deformed surface is acquired by scanning the exposed cortical surface using LRS. Both the initial surface and the deformed surface are represented by the point sets. To use the point based method presented in this paper, we firstly need to transform the two surfaces into a same space.



Figure 2. Tracked LRS (a) is positioned near the patient (b) to scan the exposed cortical surface (c)

4.1 Register the LRS space with the Image space

There are three coordinate spaces related with IGNS and two spaces related with the tracked LRS. To register the LRS space with the image space, we will use the other three coordinates spaces as the bridge to connect these two spaces. The five spaces are:

- Image space: the space defined by the image data
- LRS space: the space defined by the LRS
- Polaris space: the space defined by the Polaris
- Reference frame space: the space defined by the reference frame, which is fixed on the head of the patient and can be tracked by the Polaris
- Tracked tool space: the space defined by the tracked tool, which is fixed on the LRS and can be tracked by the Polaris

We need to transform the deformed cortical surface, which is acquired in LRS space, into the Image space.

The transformation is shown in Figure 3. Firstly, transform the LRS space to the Tracked tool space based on the calibration procedure. Then, by Polaris, transform the Tracked tool space to the Polaris space and transform Polaris space to Reference frame space. This is easily done because both the reference frame and the tracked tool can be tracked by the Polaris. At last, transform the Reference frame space to the Image space based on the routine PBR procedure in IGNS.



Figure 3. Spaces transformation

Define $T_{LRS-Img}$ as the transform from the LRS space to the Image space, $T_{LRS-Tool}$ as the transform from the LRS space to the tracked tool space, $T_{Tool-Polaris}$ as the transform from the Tracked tool space to the Polaris space, $T_{Polaris-Ref}$ as the transform from the Polaris space to the Reference frame space, $T_{Ref-Img}$ as the transform from the Reference frame space to the Image space. The transform from the LRS space to the Image space $T_{LRS-Img}$ can be expressed as:

$$T_{LRS-Img} = T_{LRS-Tool} \times T_{Tool-Polaris} \times T_{Polaris-Ref} \times T_{Ref-Img}$$
(10)

4.2 Results

Figure 4(a) is the scanned cortical surface using LRS. Figure 4(b) includes source and target point sets. We put them together to clearly show their relationship and illustrate what's partially correlated. The source point set, shown as green color points, consist of all the nodes of the mesh. The target point set include two parts shown as red color points and white color points respectively. The top red point set come from 4(a), which is transformed using equation 10. Note that LRS image includes both texture and geometric information. In this paper, we only use its geometric information i.e. the position of the point. The bottom white point set consist of fixed nodes. From Figure 4(b) we can see that the correlation only occurs at the top and the bottom. Figure 4(c) shows the deformed mesh generated by the Algorithm 1. The surface of the mesh is deformed to the scanned surface, but the bottom is still fixed as we expect. We zoom in the part of the craniotomy in Figure 4(d) to show the agreement between the scanned surface and the mesh surface. Figure 4(e) shows the deformation field.



Figure 4. The scanned surface (a) is transformed into the image space (b). (c) shows the agreement between the deformed mesh surface and the scanned surface on the top and the agreement between the deformed mesh surface and the fixed nodes at the bottom. (d) is the zoom in of the top surface in (c). (e) is the deformation field



Figure 5. preMRI (a) is deformed into (b) using the deformation field in Figure 4(e). iMRI (c) is used to compare with preMRI and deformed preMRI. (d) shows the discrepancy of the boundaries between the preMRI and the iMRI and (e) shows the discrepancy of the boundaries between the deformed preMRI and the iMRI.

Figure 5 shows the results of the qualitative evaluation of this method. Figure 5(a) is the preoperative MRI and Figure 5(b) is the deformed preoperative MRI generated by applying the deformation field shown in Figure 4(e) on the preoperative MRI. Figure 5(c) is the intra-operative MRI, which is used to evaluate the accuracy of the NRR method. In Figure 5(d), we superimpose the extracted boundary of the preoperative MRI to show the discrepancy before the NRR. After NRR, the extracted boundary of the deformed preoperative MRI will be superimposed on the intra-operative MRI. As shown in Figure 5(e), the discrepancy, especially in the vicinity of the tumor, is obviously reduced.

Figure 6 shows the quantitative evaluation of the registration method. In Figure 6, five landmarks are chosen in preMRI, deformed preMRI and iMRI respectively. The first two are superficial landmarks and the other three are deep landmarks. For each landmark, the dark blue bar is the error before registration and the light blue bar is the error after registration. The average error after registration reaches 1.164mm. In the superficial part, the average error after registration is 1.205mm and in the deep part, it is 1.137mm. The deep part has a higher absolute accuracy, but its relative accuracy(34%) is lower than that in the superficial part(78%). This is because the selected superficial landmarks approach the cortical surface, which has larger deformation, but most of the deformation can be corrected.



Figure 6. Quantitative evaluation

5. CONCLUSION

This paper presents a novel point based NRR by: one, combining biomechanical model with class RPM framework to deal with sparse point sets and second combining Least Trimmed Square with search range to deal with partially correlated point sets. The advantages of our method are two: (1) it does not rely on specific algorithms to find the correspondence in contrast to work in,^{2–4} (2) in comparison to traditional RPM method, it has more relaxed requirements for the input due to its support for the sparse and partially correlated point sets. In summary, our method can effectively address the following problem: Given sparse and even partially correlated point sets, find a mapping function. Brain shift is a typical application for this kind of input. Our preliminary experimental evaluation shows the effectiveness of this method to compensate for brain shift.

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