Scalable Quantum Edge Detection Method for D-NISQ Imaging Simulations: Use Cases from Nuclear Physics and Medical Image Computing

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Introduction

Edge Detection is one of the computationally intensive modules in image analysis. It is used to find important landmarks by identifying a significant change (or "edge") between pixels and voxels. We present a hybrid Quantum Edge Detection method by improving three aspects of an existing widely referenced implementation, which for our use cases generates incomprehensible results for the type and size of images we are required to process. Our contributions are in the pre-and post-processing (i.e., classical phase) and a quantum edge detection circuit: (1) we use space-filling curves to eliminate image artifacts introduced by the image decomposition, which is required to utilize D-NISQ (Distributed Noisy Intermediate-Scale Quantum) model; (2) we introduce a new decrement permutation circuit and relevant optimizations for mapping realistic images on today's noise Quantum Processor Unites (QPU); (3) we can improve the encoding circuit fidelity to approximately 70%, preserve the edge detection circuit fidelity to approximately 50% from <10%, and reduce the number of CX gates by approximately 68% to under 100, by using a moderate number of 128 cores for 5-qubit QPUs in the D-NISQ simulations, which are enhanced with realistic noise model from IBM. An evaluation of MRI (Magnetic Resonance Imaging) is underway, and we will report our findings.

Overview

An image is decomposed to a 1D input vector $\vec{f} = (f_0, f_1, \dots, f_{2^n-1})^T$ and transformed into the normalized quantum state $|f\rangle = (c_0, c_1, \dots, c_{2^n-1})^T$ via a series of unitary operations on qubits q_1 through q_{n+1}. The Quantum Hadamard Edge Detection circuit takes this input vector performs a series of amplitude permutations of the form:

$$(I_{2^n} \otimes H) D_{2^{n+1}} (I_{2^n} \otimes H)|f\rangle = \frac{1}{2}(c_0 + c_1, c_0 - c_1, \dots, c_{2^n-1} - c_0)^T \quad (1)$$

Where the odd indexed states take the form $|q_{2^{n+1}-1} \dots q_2\, q_1 1\rangle$ with the least significant bit being in the $|1\rangle$ state. This gives the resultant image gradient:

$$|g\rangle := \frac{1}{2}(c_0 - c_1, c_1 - c_2 \dots, c_{2^n-1} - c_0)^T \quad (2)$$

It can be noted that the original 2D to 1D decomposition can determine which pixels are measured against each other.

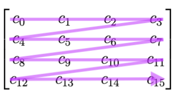

Fig. 1: Row Major Ordering decomposition provides the horizontal detection as suggested by the publishers. It can be seen how these methods of indexing will provide errors across the boundaries of the image/sub images due to non-adjacent pixel comparisons (e.g. $c_3 - c_4$) by eq. [2].

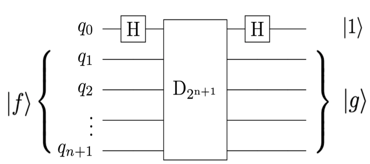

Fig. 2: Proposed QHED Circuit

An $n+1$ decrement gate using $n+1$ qubits takes a series of multi controlled CNOT (MCX) gates:

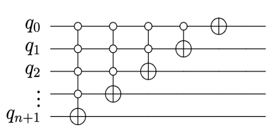

Fig. 3: Decrement Permutation without ancillary qubits

NISQ Hardware cannot directly entangle multiple qubits. We find that this construction requires an exponential number of CNOT gates in order to propagate quantum information across the qubits.

Methodology

The locality preservation for the 2D to 1D mapping can be rectified by the use space filling curves for our and/or the introduction of buffer pixels with the cost of redundancy.

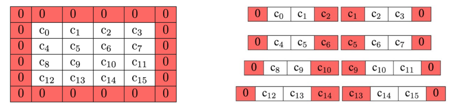

Fig. 4: Buffer pixels of value 0 are applied to the boundary of the overall image. Further decomposition is then possible by adding neighboring pixels to be used as a buffer due to QHED error. The red cells are ultimately disposed in the final output.

An alteration to the construction of the Decrement unitary for some n qubit amplitude encoding circuit to produce the desired $n+1$ output requires an additional $n-1$ ancillary qubits for a total of $2n$ qubits to create a decrement permutation which relies on only CX and Toffoli gates.

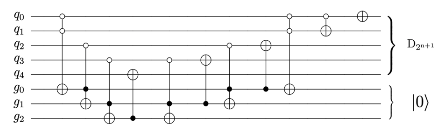

Fig. 5: $n = 4$ Decrement Permutation alternative. The open controls are activated where the qubits are in the $|0\rangle$ state, the closed controls are activated when the qubits are in the state $|1\rangle$.

In total, we may optimize the QHED circuit by the following:

- An alteration to the Decrement gate which utilizes only CNOT and Toffoli gates.
- The use of space filling curves and/or buffer cells for the decomposition.
- IBM's Qiskit Transpiler rewrites the input circuit to match a specified backend topology. In addition, it can be used to stochastically optimize a given circuit to minimize the number of two qubit CNOT operations.
- For the amplitude encoding portions of the circuit, we may use a generalized means of achieving an optimal low noise sub-graph for mapping the backend topology called *Mapomatic*.
- Measurement error can be mitigated due to experimental hardware results which ascertain a readout error matrix *A* such that:

$$\vec{P}_{noise} = A\vec{P}_{ideal}$$

We use a heuristic fidelity estimate from IBM to determine the optimization effects of the circuits:

$$f_{circuit} = f_{1Q}^{N_{1Q}} f_{2Q}^{N_{2Q}}$$

Where f_{1Q} and f_{2Q} are the average fidelity estimates for single and two qubit gates for a particular backend with N_{1Q} and N_{2Q} being the respective number of each category of gates per circuit.

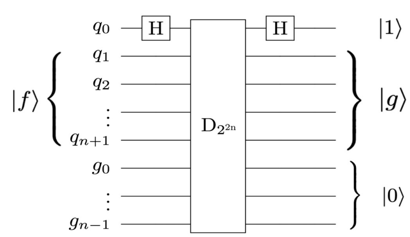

Fig. 6: Resultant QHED Circuit with the proposed Decrement gate with ancillary qubits

Results

We encode a particle image onto a simulated noisy IBM hardware backend with an increasing number of n qubits per domain where each domain contains 2^n pixels of the image, the results are averaged across all domains to demonstrate the limitations of amplitude encoded circuits. We build the respective QHED circuits of each domain size displaying the effects of our modifications:

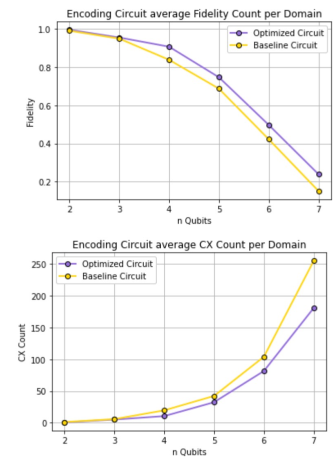

Fig. 7: Amplitude encoding circuit for a binary particle trace image broken into domain sizes of *n* qubits. Metrics are average across all circuits for each domain.

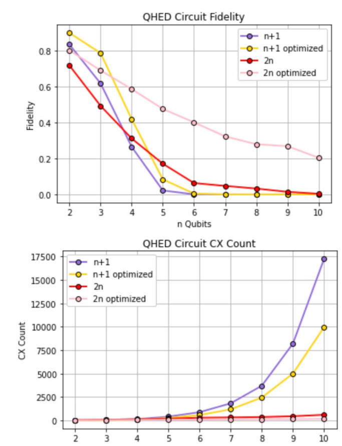

Fig. 8 QHED circuit metrics divided by n qubit domain sizes

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