# PARALLEL CONSTRAINED DELAUNAY MESHING ALGORITHM IN THREE DIMENSIONS

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## ABSTRACT

Mesh generation is an integral part of the finite element method and as such it has to be scalable and efficient in order to keep the throughput of the simulation pipeline high. We present an initial design of a three dimensional Parallel Constrained Delaunay Meshing Algorithm that uses mesh decomposition techniques to create work units to be executed in parallel and existing sequential meshing software to refine them (ie, maximum possible code re-use). Asynchronous messages are employed to achieve conformity between shared regions of the work units while keeping the overhead low.

**Keywords:** parallel mesh generation, Delaunay, asynchronous

## 1 INTRODUCTION

In our previous work (Chernikov and Chrisochoides 2008) we presented an algorithm for parallel constrained Delaunay mesh generation in two dimensions called Parallel Constrained Mesh Generation algorithm (PCDM). PCDM employs two-dimensional domain decomposition techniques to divide the original problem into smaller subdomains. Every subdomain is meshed separately by a different process. Neighboring subdomains exchange asynchronous messages containing information related to the splits of shared segments necessary to achieve conformity on shared interfaces. The asynchronous nature of the messages together with their small size introduce a small overhead to PCDM allowing the method to scale up linearly up to 100 processes.

In this work we present, PCDM3D, a design of the three dimensional version of our previous two dimensional algorithm. PCDM3D starts with an existing mesh and refines it in parallel while keeping the triangulation of the shared interfaces the same across all subdomains resulting thus in a mesh which is Constrained Delaunay per subdomain and consistent across the interfaces. Although, PCDM3D shares many ideas with our previous method, it has also to deal with new issues. In particular, (a) the refinement rules for three dimensional geometries are more complex and depend on the features of the input, (b) points can now be inserted both on faces and segments and finally (c) interfaces can be shared by more than two subdomains.

Moving to three dimensions introduces many new challenges. First, domain decomposition of arbitrary three dimensional geometries is not trivial and there are not known methods for the general case. We need thus to provide an alternative input for the mesher. Second, the meshing method that we use (Constrained Delaunay refinement) is more computationally challenging in three dimensions and can often require more

*SpringSim-ANSS, 2017 April 20, Suffolk, VA, USA;*

than one points in order to split a single element while respecting the invariant properties of the mesh (Si and Shewchuk 2014). Finally, the concurrent access on the interface triangulation may create non-conforming interfaces since it is known that the order of processing the elements on a triangulation can result in a different triangulation.

## 2 METHODS

To avoid the complexities introduced by small angles in three dimensional objects we will assume for the rest of the paper that the input satisfies the projection condition as described in (Shewchuk 1997). This condition guarantees that the meshing algorithm will terminate and it will produce a mesh of a certain quality while inserting one point at a time. In practice, to achieve the requirements of the projection condition we will use surface meshes with dihedral angles of 90 degrees obtained from a hexahedral mesh composed entirely by cubes. Having right angles throughout the mesh takes away the need for high quality partitions since any partition will have 90 degrees angles. This enables us to use generic graph partitioning software METIS (Karypis and Kumar 2016) for decomposing the mesh into subdomains which are then distributed to the processes for sequential refinement using TetGen (Si 2013).

Concurrent access of the shared boundary was not an issue in our previous work because in two dimensions the triangulation of the interface is just the discretization of the edges and since the points are inserted according to theory on predefined positions, conformity can be achieved by making sure that all the points have been inserted. On the other hand, in three dimensions we need to enforce the same order of processing the elements on the boundary. To achieve this a token will be assigned to each interface. The token will be unique and only one subdomain can own it at each point of time. Only the subdomain holding the token is allowed to modify the interface. As soon as the subdomain inserts a point on the shared interface it has to send the token to the next of the neighbors and when the token completes a cycle, it means that the modifications on the interface have been synchronized. The token can also be requested by a subdomain if it is not present. In this way all the subdomains will have a chance to insert points on the interface.

## 3 FUTURE WORK

The three dimensional version of PCDM is currently under active development. The loose coupling between the subdomains is expected to allow the algorithm to scale up reasonably well with low communication overhead. We also plan to formalize and prove the correctness of the discussed communication scheme.

## 4 ACKNOWLEDGMENTS

This work in part is funded by the Modeling and Simulation fellowship, NSF grant no. CCF-1439079, NASA grant no. NNX15AU39A and DoD's PETTT Project PP-CFD-KY07-007.

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